

A **finite interpretation** for a set of sentences from PredLog consists of the following:

- A finite universe (or domain) U .
- Extensions for all the predicates appearing in the sentences.
- Extensions for the names appearing in the sentences.
- Truth value specifications for the sentence letters appearing in the sentences.

The truth values of (complex) sentences of PredLog in a given interpretation are determined as follows:

- Formulas of the form $F\alpha$ are true iff the extension of the name α is an element of the extension of the predicate F .
- Formulas of the form $F^n\alpha_1\dots\alpha_n$ are true iff the ordered n -tuple of the extensions of the names $\alpha_1\dots\alpha_n$ is an element of the extension of the predicate F .
- Formulas of the form $\alpha=\beta$ are true iff the extension of the name α is the same as the extension of the name β .
- The truth conditions for connectives (\wedge , \rightarrow etc.) are the same as in PropLog.
- Quantified sentences: Construct the expansion of all universal and existential formulas, then assign truth values for the resulting quantifier-free sentences according to rules above. The truth value of a quantified sentence (relative to U) is the truth value of its expansion.

The **expansion of a universal wff** relative to a universe of n elements consists of n conjuncts, where the n th conjunct is an instance of the formula with the name of the n th element in the universe as the instancial name. (We refer to this as a **universal expansion** for short.)

Example.

The expansion of

$$\forall x(Fx \rightarrow Gx)$$

in the universe $\{a\}$ is

$$(Fa \rightarrow Ga).$$

In the universe $\{a,b\}$ its expansion is

$$(Fa \rightarrow Ga) \wedge (Fb \rightarrow Gb).$$

In the universe $\{a,b,c\}$ its expansion is

$$(Fa \rightarrow Ga) \wedge (Fb \rightarrow Gb) \wedge (Fc \rightarrow Gc),$$

and so on.

The **expansion of an existential wff** relative to a universe of n elements consists of n disjuncts, where the n th disjunct is an instance of the formula with the name of the n th element in the universe as the instancial name. (We refer to this as an **existential expansion** for short.)

Example.

The existential

$$\exists x(Fx \wedge Gx)$$

expands to

$$(Fa \wedge Ga) \vee (Fb \wedge Gb) \vee \dots$$

for the universe $\{a,b, \dots\}$.

In cases of overlapping scopes of quantifiers, always expand the quantifier with the wider scope first!

Example

$\forall xFx \rightarrow \forall xGx \vdash p \rightarrow \exists xGx$ is false (the inference is invalid).

$\forall xFx \rightarrow \forall xGx \vdash p \rightarrow \exists xGx$ is false (the sequent cannot be derived)

Proof (by giving a countermodel of the inference):

Interpretation:

$U: \{a,b\}$ $F: \{a\}$ $G: \{ \}$ p is true

Expansions:

$\forall xFx$ expands to $Fa \wedge Fb$

$\forall xGx$ expands to $Ga \wedge Gb$

$\exists xGx$ expands to $Ga \vee Gb$

premise: $Fa \wedge Fb \rightarrow Ga \wedge Gb$

$T \wedge F \rightarrow F \wedge F$

$F \rightarrow F$

T

conclusion: $p \rightarrow (Ga \vee Gb)$

$T \rightarrow (F \vee F)$

$T \rightarrow F$

F