assumption Assume any sentence.
Annotation: A
Assumption set: The current line number.

1	(1)	p∨q	**A**

roof-intro Given two sentences (at lines *m* and *n*), conclude a conjunction of them.
Annotation: **m,n ∧I**
Assumption set: The union of the assumption sets at lines *m* and *n*.
Comment: The order of lines *m* and *n* in the proof is irrelevant. The lines referred to by *m* and *n* may also be the same.

1	(1)	p	A
2	(2)	q	A
1,2	(3)	p∧q	1,2 ∧I

roof-elim given a sentence that is a conjunction (at line *m*), conclude either conjunct.
Annotation: **m ∧E**
Assumption set: The same as at line *m*.

1	(1)	p∧q	A
1	(2)	q	1 ∧E
1	(3)	p	1 ∧E

wedge-intro Given a sentence (at line *m*), conclude any disjunction having it as a disjunct.
Annotation: **m ∨I**
Assumption set: The same as at line *m*.

1	(1)	p	A
1	(2)	p∨q	1 ∨I

wedge-elim Given a sentence (at line *m*) that is a disjunction and another sentence (at line *n*) that is a denial of one of its disjuncts conclude the other disjunct.
Annotation: **m,n ∨E**
Assumption set: The union of the assumption sets at lines *m* and *n*.
Comment: The order of m and n in the proof is irrelevant.

1	(1)	p∨q	A
2	(2)	¬p	A
1,2	(3)	q	1,2 ∨E

arrow-intro Given a sentence (at line *n*), conclude a conditional having it as the consequent and whose antecedent appears in the proof as an assumption (at line *m*).
Annotation: **n →I(m)**
Assumption set: Everything in the assumption set at line *n* excepting *m*, the line number where the antecedent was assumed.
Comment: The antecedent must be present in the proof as an assumption. We speak of discharging this assumption when applying this rule. Placing the number m in parentheses indicates it is the discharged assumption. The lines m and n may be the same.

1	(1)	¬p∨q	A
2	(2)	p	A
1,2	(3)	q	1,2 ∨E
1	(4)	p→q	3 →I(2)

arrow-elim Given a conditional sentence (at line *m*) and another sentence that is its antecedent (at line *n*), conclude the consequent of the conditional.
Annotation: **m,n →E**
Assumption set: The union of the assumption sets at lines *m* and *n*.
Comment: The order of *m* and *n* in the proof is irrelevant.
Also known as: Modus Ponendo Ponens (MPP), Modus Ponens (MP), Detachment, Affirming the Antecedent.

1	(1)	p→q	A
2	(2)	p	A
1,2	(3)	q	1,2 →E

reductio ad absurdum Given both a sentence and its denial (at lines *m* and *n*), conclude the denial of any assumption appearing in the proof (at line *k*).
Annotation: **m,n RAA(k)**
Assumption set: The union of the assumption sets at *m* and *n*, excluding *k* (the denied assumption).
Comment: The sentence at line *k* is the assumption discharged (a.k.a. the **reductio assumption**) and the conclusion must be a denial of the discharged assumption. The sentences at lines *m* and *n* must be denials of each other.
Also known as: Indirect Proof (IP), ¬Intro/¬Elim.

1	(1)	p→q	A
2	(2)	¬q	A
3	(3)	p	A
1,3	(4)	q	1,3 →E
1,2	(5)	¬p	2,4 RAA(3)

double-arrow-intro Given two conditional sentences having the forms ϕ→ψ and ψ→ϕ (at lines *m* and *n*), conclude a biconditional with ϕ on one side and ψ on the other.
Annotation: **m,n ↔I**
Assumption set: The union of the assumption sets at lines *m* and *n*.
Comment: The order of *m* and *n* in the proof is irrelevant.

1	(1)	p→q	A
2	(2)	q→p	A
1,2	(3)	p↔q	1,2 ↔I

double-arrow-elim Given a biconditional sentence ϕ↔ψ (at line *m*), conclude either ϕ→ψ or ψ→ϕ.
Annotation: **m ↔E**
Assumption set: The same as at *m*.

1	(1)	p↔q	A
1	(2)	p→q	1 ↔E
1	(3)	q→p	1 ↔E